Acoustic quasimodes in two-dimensional dispersed random media

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Using the generalized coherent-potential-approximation approach, we present the dispersion relation of the two-dimensional dispersed random media. In the intermediate-frequency regime, two acoustic modes are found in colloidal suspensions including cylindrical plastic rod in water background. The scattering cross section offers a good explanation for the two modes and the observed frequency gaps in the excitation spectra.

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I. INTRODUCTION

In recent years, there has been growing interest in classical wave propagation and scattering in composite media [1–18]. For periodic media, it is concerned with the band gap in the dispersion relations of photonic or phononic crystals [2–8]. For the random media, the dispersion relation also provides rich physics [9–18]. If the wavelength is comparable to the scale of random inhomogeneities, the character of the wave propagation is generally expected to change drastically due to the strong multiple scattering.

It is conventional wisdom that an isotropic, homogeneous, elastic solid has one longitudinal and two transverse modes, and a fluid has only one longitudinal acoustic mode. For the fluid-saturated porous media, where the solid and the fluid phase form a connected network, experiments have confirmed Biot's prediction that it exists as two longitudinal modes in addition to the two shear modes, where the fast longitudinal mode travels predominantly in the solid frame and the slow one travels mainly in the fluid [9]. For the colloidal suspensions, consisting of monodisperse PMMA spheres with a certain size dispersed in oil, both experiment and theoretical calculations show that in the intermediate-frequency regime, there exist two distinct longitudinal modes with finite lifetimes [11–13]. The existence of a propagating mode put forward a challenge to the conventional understanding that only diffusive transport exists in the strong-scattering regime.

In this paper, we develop a generalized coherent-potential-approximation (GCPA) approach for the identification of quasimodes in two-dimensional dispersion random media consisting of random parallel solid rods immersed in fluid. The dispersion relations of three kinds of systems are discussed. In the intermediate-frequency regime where the wavelength is comparable to the scale of the inhomogeneities, significant phenomena are found. Two acoustic modes are surprisingly found in colloidal suspensions including a cylindrical plastic rod in water background, which arises from the coherent coupling of resonances on neighboring particles. Furthermore, calculated results of the cross section provide us with an excellent prototype for further understanding the physical origin of the two acoustic modes in the plastic-water system and the observed frequency gaps in the dispersion relations. In order to make this more clear, the details of the generalized GCPA approach theory are outlined in Sec. II. Section III presents the results and discussion and Sec. IV summarizes this paper.

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II. METHOD

The two-dimensional dispersed random system studied is characterized by the dispersion microgeometry where each solid cylinder is individually enveloped by the fluid but all the cylinders are aligned parallel to one another. We discuss the waves propagating in the two-dimensional plane that are vertical to the cylinders' axis. The configuration of the system is illustrated in Fig. 1.

To calculate the effective macroscopic properties of the system, we employ the effective medium model [1], which is schematically depicted in Fig. 2. The coated cylinder is embedded in a homogenized effective medium composed of similar units of coated cylinders, with the effective-medium speed of the medium to be determined by some self-consistent condition. In the coherent-potential approximation (CPA), such a condition is achieved by requiring the vanishing of the forward-scattering amplitude $f(0)$ through the adjustment of the effective-medium wave speed C_e. The condition is noted to be self-consistent in the following sense: If we let G denote the exact Green's function for an acoustic wave in the random system, then

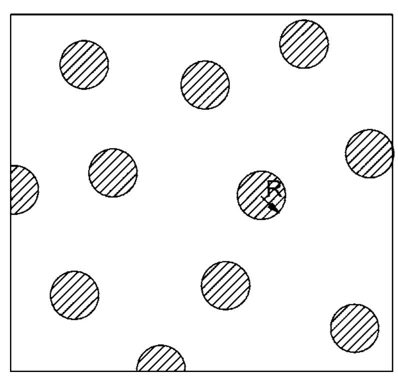

FIG. 1. The configuration of the system consisting of parallel solid rods randomly immersed in fluid.

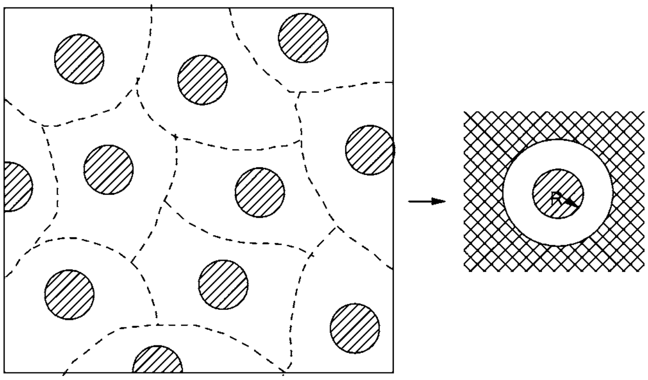

FIG. 2. The effective medium model. The coated cylinder is embedded in a homogenized effective medium composed of similar units of coated cylinders.

$$G = G_e + G_e T G_e,$$

where G_e is the Green's function for the homogeneous effective medium and T denotes the exact total scattering operator including all the multiple scattering between particles. By expressing

$$G_e = \frac{1}{p^2 - q^2}, \quad (1)$$

where $q=\omega/C_e$, C_e is the effective-medium wave speed and p is the Fourier-transform variable, the CPA condition for Eq. (1) is $\langle T\rangle=0$ through the adjustment of q, where the angular brackets denote configuration average. When that happens, $\langle G\rangle=G_e$ and q is identified as the wave vector of the excitation. Since $\langle T\rangle=nt$ in the weak-scattering limit, where t is the single-coated-cylinder forward-scattering amplitude and n the cylinder density, $f(0)=T_{qq}=0$ is therefore the condition for determining q. $\langle T\rangle=0$ is equivalent to $t=0$, which means the CPA condition is consistent with requiring the forward scattering.

For the generalized CPA condition, instead of requiring $\langle T\rangle=0$, we look for minima of $\langle T\rangle$. The fact that the scattering now does not vanish on average means that the excitation must be a quasimode. However, since at minima the scattering may still be weak, we may approximate $\langle G\rangle$ by

$$\langle G\rangle \approx \frac{1}{p^2 - q^2 - \Sigma}$$

where the self-energy $\Sigma \approx \langle T\rangle \approx nt$ to the first order in scattering strength, The minima of $\langle T\rangle$ may thus be identified by the maxima of density of state (S_D),

$$S_D(q,\omega) = -\frac{1}{\pi}\,\mathrm{Im}\langle G\rangle,$$

which is evaluated with the condition of elastic scattering ($p=q$), so that

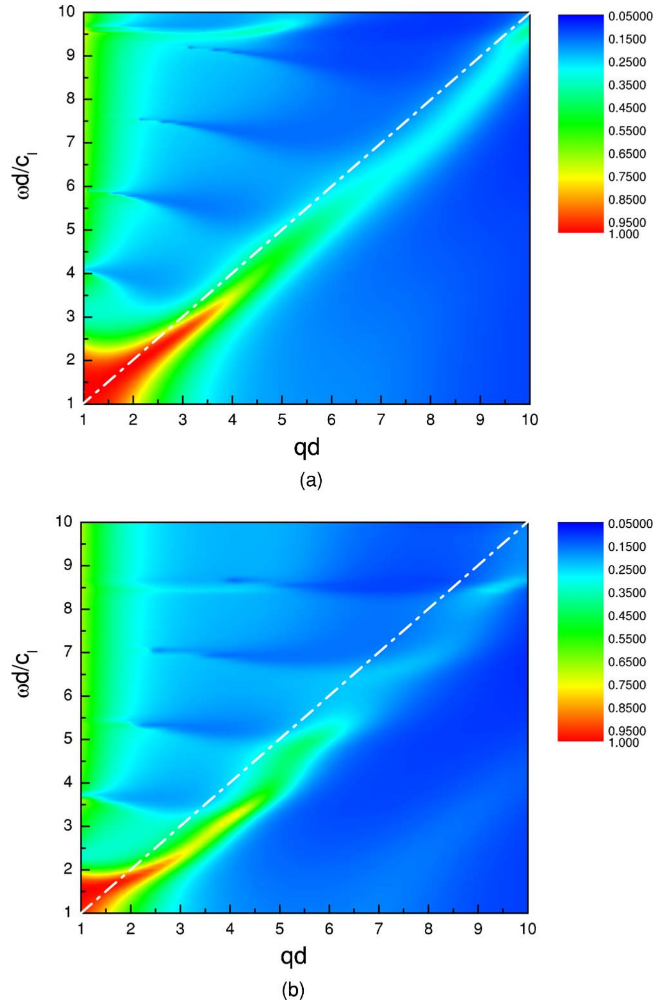

FIG. 3. (Color online) The S_D plotted as a function of the normalized frequency $\omega d/c_1$ and wave vector qd, where c_1 is the longitudinal velocity for the fluid background. The magnitude is indicated by the colors, with the scale bar showing the values from low to high corresponding to the colors. The dispersion falls below the dispersion curve for pure water (dashed white line). The volume fraction of the cylinders is 0.5. (a) Glass cylinders in water and (b) Fe cylinders in water.

$$\langle G\rangle \approx -\frac{1}{nt}.$$

The maxima of S_D correspond directly to the minima in scattering, which give the best condition for the existence of a quasimode since less scattering means the wave can coherently propagate over a longer distance. In the following sections we give details and justifications for the results stated in this section.

III. RESULTS AND DISCUSSIONS

We first consider the composite systems consisting of parallel glass cylinders dispersed randomly in the water background and parallel Fe cylinders dispersed randomly in the water background. In Figs. 3(a) and 3(b), we plot, in color, the calculated S_D as a function of the dimensionless fre-

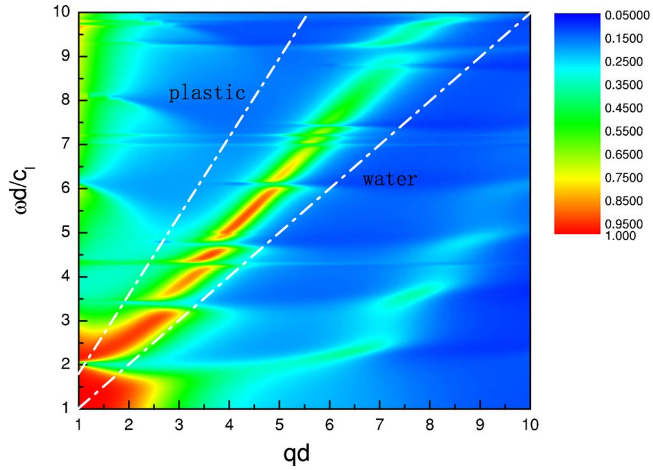

FIG. 4. (Color online) The S_D for plastic cylinders in water. The volume fraction of the cylinders is 0.5. The two dashed white lines indicate the dispersion curve for the compressional wave in the pure liquid phase (water) and for the longitudinal wave in the solid phase (plastic), respectively.

quency $\omega d/c_1$ and dimensionless wave vector qd for the solid cylinder's volume fraction $\Phi=0.5$ for these two systems, respectively. The volume fraction Φ determines the liquid-coating thickness used in the calculation. The material parameters are chosen as follows: $\rho=7.8\ \mathrm{g/cm^3}$, $C_1=5.7\ \mathrm{km/s}$, and $C_t=3.0\ \mathrm{km/s}$ for Fe; $\rho=2.6\ \mathrm{g/cm^3}$, $C_1=5.6\ \mathrm{km/s}$, and $C_t=3.4\ \mathrm{km/s}$ for glass; $\rho=1.0\ \mathrm{g/cm^3}$ and $C_1=1.5\ \mathrm{km/s}$ for water; where ρ, C_1, and C_t are, respectively, the density, the longitudinal, and the transverse sound velocity. Significant dispersions are observed. There is only one acoustic mode indicated by the outstanding colors. The dispersion curve, defined by the peaks, is accurately determined because the widths of the peaks are substantially less than their central frequencies. It should be stressed that the calculation has no adjustable parameter. Remarkably, for most of the frequency we considered both velocities are substantially less than the speed of sound in either the longitudinal velocity in water or the longitudinal or transverse velocity in the glass.

In Fig. 4, we plot the S_D for parallel plastic cylinders dispersed randomly in the water background with the volume fraction of the cylinders 0.5. The material parameters are chosen as $\rho=1.0\ \mathrm{g/cm^3}$, $C_1=2.7\ \mathrm{km/s}$, and $C_t=1.1\ \mathrm{km/s}$ for plastic. Two bands of ridges are clearly seen in green. It should be noted that the value of qd extends from much smaller than 1 to 10, i.e., the wavelength of the acoustic excitations extends continuously from being very much larger than the cylinder diameter to being much smaller that that scale. The two observed modes are quasimodes with a limited lifetime. We see one dispersion relation exists between the plastic dispersion relation and the water dispersion relation, which are marked by white dashed lines. That means the high-frequency mode has an intermediate velocity between those of the plastic and water. These tendencies are in accord with general intuition. While not shown, our calculations also indicate that at high q high-frequency mode tends to converge to the fluid dispersion relation. [Moreover, small gaps are found in the dispersion relation relative to the high-frequency mode.] The other dispersion relation for the low-frequency mode is seen to fall below that of the liquid background. In the low frequency regime, the velocities of the low-frequency mode are less than the speed of sound in the longitudinal velocity in water. Lots of the numerical results for the solid-fluid system show that when the density of the cylinder is comparable to the density of fluid, and the longitudinal sound velocity of the cylinder is a little larger than that of fluid, there exist two quasimodes.

In Fig. 5, we plot the S_D for plastic cylinders immersed in water with different volume fractions of the cylinders Φ. We can see that as the volume fractions Φ increase, the velocity

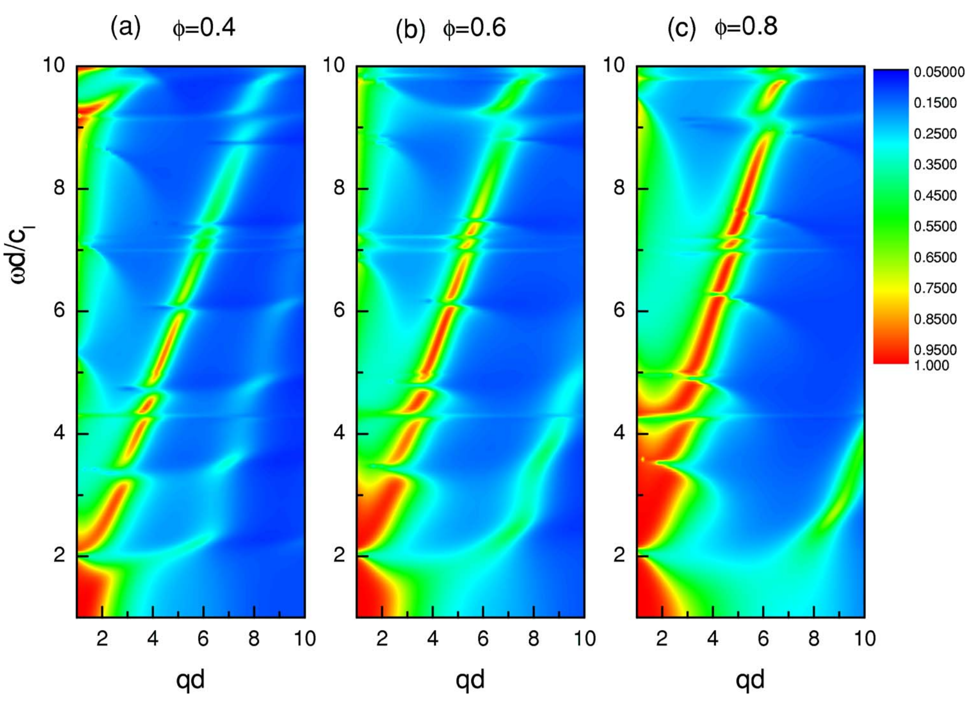

FIG. 5. (Color online) The S_D for plastic cylinders in water. The volume fraction of the plastic cylinder Φ, (a) 0.4, (b) 0.6, and (c) 0.8.

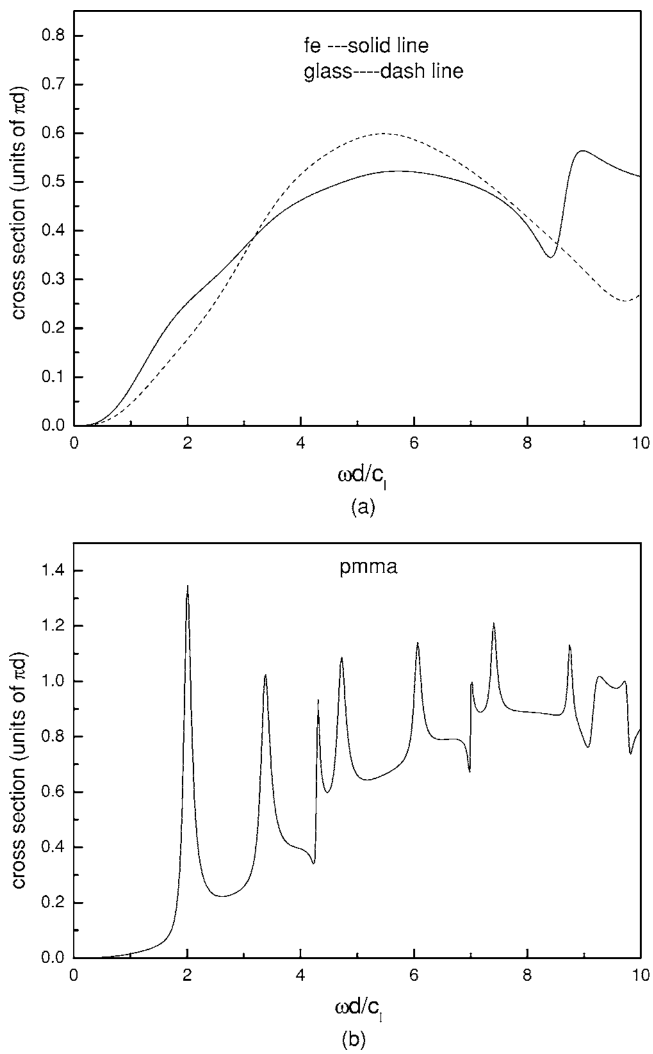

FIG. 6. The renormalized scattering cross section plotted as a function of frequency $\omega d/c_1$. (a) A Fe cylinder and a glass cylinder, respectively, are immersed in water. (b) A plastic cylinder is immersed in water.

for the high-frequency mode increases, while the low-frequency mode slows down, which is just the opposite to the trend of the high-frequency modes. At low volume, the two dispersion relations will merge and become indistinguishable from the fluid dispersion.

Furthermore, we demonstrate the behaviors of the scattering cross section in Fig. 6. It is the key to understanding the physical origin of the two modes. Scattering cross sections are renormalized in units of πd. Comparing the scattering cross section of a single Fe cylinder or a single glass cylinder immersed in water [Fig. 6(a)] with that of a single plastic cylinder immersed in water [Fig. 6(b)], we can see that in the regime of renormalized frequency from 1 to 10, there are many peaks in the scattering cross section for the plastic-water system, which means the strong resonance scattering of single cylinder. It results in the resonance on neighboring cylinders coupling through their decaying portions in the liquid, causing a splitting of each peak into two, with a minimum in between. It is this minimum that is picked up as the peak in the S_D, which corresponds to the low-frequency mode. The frequency positions of the peaks in the scattering cross section for a single plastic cylinder immersed in the water background are found to correspond directly to the gaps in the dispersion relation of the high-frequency mode. This correspondence suggests that the high-frequency mode results from the antiresonance of a single cylinder, where the scattering is minimum. Above all, the existence of two different modes depends on the single cylinder's strong resonance scattering.

IV. CONCLUSIONS

In conclusion, we investigate acoustic wave propagating in two-dimensional dispersion disorder media consisting of random parallel solid rods immersed in water. By using the GCPA approach based on the principle of locating the minima of $\langle T \rangle$, the quasimodes and dispersion relations of the Fe-water system, glass-water system, and plastic-water system are investigated. The results of our study surprisingly show significant phenomena of wave transport in the intermediate-frequency regime where the wavelength is comparable to the scale of the inhomogeneities. Two acoustic quasimodes exist in plastic-water systems, which is contrary to the conventional view that only diffusive transport exists in the strong scattering regime. Their characteristics are shown to vary with the concentration of the solid cylinders. A cross section of these kinds of systems offers a good explanation for the two modes and the observed frequency gaps in the excitation spectra.

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